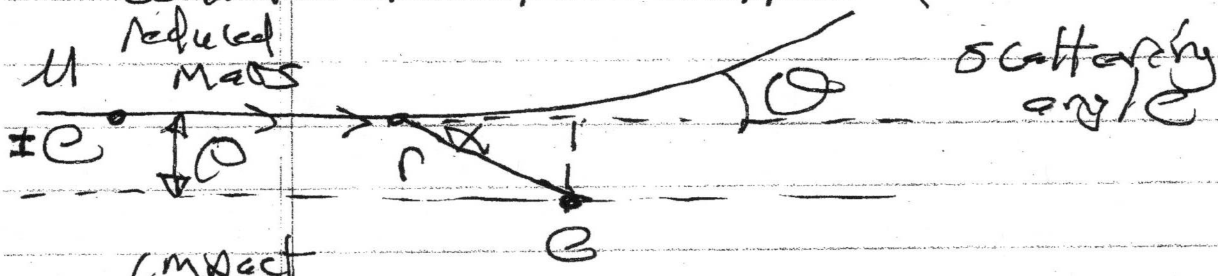
→ Basics of Collisions — ⟨ Long Range Coulomb Interaction

Transport ⟷ Coulomb Collisions vs hard sphere

→ Consider familiar collision:



— what is cross section?

in particular, seek cross section for weak deflection ⟷ "momentum transfer cross-section"

i.e. More glancing collisions occur...

— of course, central force, so $|p|$ conserved, but direction changed

deflection

$$\mu \Delta V_\perp = \Delta p_\perp = \int_{-\infty}^{+\infty} dt\, F_\perp$$

$$= \int_{-\infty}^{+\infty} dt\, \frac{e^2 \sin\alpha}{r^2}$$

$$= \int_{-\infty}^{+\infty} dt\, \frac{e^2}{r^2}\frac{\rho}{r}$$

$$r^2 = \rho^2 + v^2 t^2$$

$$\Delta p_\perp = e^2 \int_{-\infty}^{+\infty} \frac{\rho\, dt}{(\rho^2 + v^2 t^2)^{3/2}} \sim e^2 \rho \int_{-\infty}^{+\infty} \frac{1}{\rho^3} \frac{dt}{\left(1 + \frac{v^2 t^2}{\rho^2}\right)^{3/2}}$$

$$\sim e^2/\rho v$$

but $\Delta p_\perp \sim \mu v \sin\theta$
$\sim \mu v \theta$ (weak) defn.

so deflection angle:

$$\boxed{\theta \sim e^2/\mu v^2 \rho}$$

$\rightarrow$ net deflection χ

then for cross section:

$$d\sigma = \underbrace{\rho\, d\rho}_{\text{area of interaction cylinder}} = d(\rho^2) = d\left(e^2/\mu v^2 \theta\right)^2$$

i.e. note key point: cross section heavily weights weak deflections

$$d\sigma \cong \left(\frac{e^2}{\mu v^2}\right)^2 \frac{d\theta}{\theta^3}$$

most of scatt. soft → weak momentum transfer.

Now: $d\sigma = \left(\frac{e^2}{\mu v^2}\right)^2 \frac{d\theta}{\theta^3}$

infrared divergence

weak deflection divergence

$\Delta p \sim e^2/\rho v$

n.b.: small θ ↔ large ρ

⇒ long range character of Coulomb force!

low Δp, long range.

⇒ screening, long range cut-off is very relevant.

Now for momentum transfer cross-section need take out [illegible] collisions with no transfer, i.e.

$d\sigma_t = (1-\cos\theta)\, d\sigma$

$\simeq \theta^2 \left(\frac{e^3}{\mu v^2}\right)^2 \frac{1}{\theta^3}$

so $d\sigma_t \simeq \left(\frac{e^2}{\mu v^2}\right)^2 \frac{1}{\theta}$

$$\therefore \quad \boxed{\sigma_f \simeq \left(\frac{e^2}{\mu v^2}\right)^2 \ln\left(1/\theta_0\right)}$$

divergence - low θ.

- Coulomb cross-section, ∴ Rutherford.
- θ_0 is small angle cut-off

now low $\theta \Leftrightarrow$ large ρ

$\Rightarrow$ small angle cut-off set by large ρ

∴ largest ρ can be $\lambda_D \leftarrow$ screening limited!

Now,

$$\theta \sim e^2/\mu v^2 \rho$$

so

$$\boxed{\theta_0 \sim e^2/\mu v^2 \lambda_D}$$

screening cut-off

So $\ln \Lambda = \ln(1/\theta_0) = \ln(T\lambda_D/e^2)$

Λ: L (in LL)

$\ln(T\lambda_D/e^2)$: Coulomb Logarithm [can resolved by $G \to L+B$]

$$\boxed{\sigma_t \simeq \left(\frac{e^2}{T}\right)^2 \ln \Lambda} \rightarrow \text{effective cross section}$$

$\Rightarrow$

$$\boxed{\sigma_t \sim \bar{r}^2 \left(e^2/\bar{r}T\right)^2 \ln \Lambda}$$

Coulomb cross section

Note: $\left(e^2/\bar{r}T\right)^2 \rightarrow \left(\left[1/n\lambda_D^3\right]^{2/3}\right)^2 \sim \dfrac{\bar{r}^4}{\lambda_D^4}$

$\left(\bar{r}^2/\lambda_D^2\right)^{-2}$

$\sim 1/\left(n\lambda_D^3\right)^{4/3}$

$$* \quad \boxed{\sigma_t \sim \bar{r}^2 \left(1/n\lambda_D^3\right)^{4/3} \ln \Lambda} \quad *$$

Now,

$$l_{mfp} \sim 1/n\sigma_T$$

$$\sim 1/n\bar{r}^2\left(\frac{e^2}{\bar{r}T}\right)^2 \ln\Lambda$$

$$\sim \bar{r}\,(n\lambda_D^3)^{4/3}/\ln\Lambda$$

$$\boxed{l_{mfp} \cong \bar{r}\,(\lambda_D/\bar{r})^4/\ln\Lambda}$$

so

$$l_{mfp} \simeq \bar{r}\,(\lambda_D/\bar{r})^4/\ln\Lambda$$

$$\frac{l_{mfp}}{\lambda_D} \simeq \left(\frac{\lambda_D}{\bar{r}}\right)^3/\ln\Lambda$$

$$\left[\sim n\lambda_D^3/\ln\Lambda\right]$$

as $n\lambda_D^3 \gg \ln\Lambda$

$$\boxed{l_{mfp} \gg \lambda_D.}$$

[consistent with screening

so

$$\boxed{\bar{r} < \lambda_D < l_{mfp} < L}$$

collisional plasma ordering

Note: $\dfrac{l_{mfp}}{\lambda_D} \cong \dfrac{\bar{r}}{\lambda_D}\left(\dfrac{\lambda_D}{\bar{r}}\right)^4 \Big/ \ln\Lambda$

$\cong \left(n\lambda_D^3\right) \Big/ \ln\Lambda$

Now, further points about transport:

- apart from $\ln\Lambda$, no mass, M scaling in σ_t, l_{mfp}.
- so $\nu_{col} \sim M^{-1/2}$, via v_{th}.
  $\tau_{col} \sim M^{1/2}$

  so $\dfrac{\tau_{e,e}}{\tau_{ii}} \sim \left(m_e/m_i\right)^{1/2} \ll 1$

then, as before (gases):

→ thermal conductivity

$\lambda \sim n \, v_{Th} \, l_{mfp}$

$n \to C_v$; $v_{Th} \to$ larger for electrons; $l_{mfp} \to$ indep μ

so

electrons control thermal conduction

→ viscosity

$\eta \sim M_i n \, v_{Th_i} \, l_{mfp}$

$l_{mfp} \sim 1/n\sigma$

ions control flow

contrast:

$\eta_e \sim M_e n \, v_{Th_e} l_{mfp}$

$\eta \sim M \, v_{Th_i} / \sigma_t$

$$\boxed{\eta \sim (M_i T)^{1/2} / \sigma_t}$$